Numerical simulation of a piano soundboard under downbearing

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Abstract
A finite element model of a piano soundboard is used to study the effect of the strings tension (downbearing) on its vibration, considering the ribs, the bridges and the crown. The downbearing is modeled with the prestress theory. Prestress calculation with linear and nonlinear models including geometric rigidity are compared in terms of the modal frequencies. The effects of the downbearing in modal frequencies and mobility are investigated and the importance of the crown on these results is evaluated. A simple phenomenological law is exhibited, which characterizes the evolution of eigenfrequencies with downbearing, including the initial crown.

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Introduction

In 1940, P. H. Bilhuber and C. A. Johnson\textsuperscript{1} tackled the question of the importance of the downbearing of the soundboards in the piano sound quality. They observed notable differences in the energy of vibration related to the downbearing level, but they did not come to a conclusion about changes in perceivable timbre due to downbearing. From that work, measurements have dealt with the soundboards properties\textsuperscript{2–8}, for different kinds of pianos, including a one-design prototype\textsuperscript{2}. T. R. Moore and S. A. Zietlow\textsuperscript{3} are the only ones who make a comparison in modal shapes for an upright piano without and with strings. They observed large changes in the first modal shapes and frequencies with the strings application. K. Wogram\textsuperscript{4} and H. A. Conklin\textsuperscript{5} showed that the soundboard mobility on the bridges depends on of the strings tension in terms of frequencies and quality factor.

Some authors have proposed models of piano soundboards. K. Wogram felt the possible use of the finite element analysis to simulate the making process. N. Giordano\textsuperscript{9} showed that a simple model made of an orthotropic plate and ribs is sufficient to simulate the global properties. J. Berthaut et al.\textsuperscript{6} chose a precise finite element model, and confirmed the performance of this method thanks to a comparison with an experimental modal analysis. But even if the downbearing was considered as one of the most important technique for the piano making, its influence has never been modeled. This paper proposes one modeling of the downbearing by using a prestress approach, including the crown amplitude.

If one considers the vibration, the prestress theory consists in attributing a geometric energy due to an external loading. The total rigidity is the sum between the classical elastic rigidity and the geometric one. Finite element softwares are able to express this total rigidity. Considering small deformation due to the wood characteristics and with the hypothesis of an elastic and linear material, the nonlinearity may come only from the change in geometry during the loading. Two approaches to include prestresses have to be decided:

- the linearized one\textsuperscript{10}. The geometric energy is simplified as a linear expression of the loading parameter. This approximation reduces the time of calculus but is limited by
the amplitude of displacement.

- the geometrically nonlinear one\textsuperscript{11}, including the hypothesis of static large displacements and small deformations of the structure. The geometric rigidity is calculated by an iterative method. The updating at each step of displacement requires much more time of calculus but is necessary if the shape of the system is greatly modified by the loading process.

This paper proposes to find the best way to model the effect of the downbearing on the soundboard properties. This model is made of a soundboard with ribs and bridges coming from a real one. The downbearing is introduced by an induced geometric rigidity. An emphasis is given to the influence of the initial crown on downbearing effects.

**Finite element model**

The finite element model is based on the soundboard of a 1.80m IBACH piano of figure 1. It is made of the plate, fifteen ribs and two bridges. The model considers precisely the geometry of the plate and the ribs, the position of the ribs and the bridges. Implemented with CAST3M software\textsuperscript{12}, the mesh is a 3742 nods and triangle elements with a kinematic of
Kirchhoff plates, modeling the whole soundboard. The material properties have been found in the literature\(^6\), and are presented in table I.

The thickness is here considered as a 8mm constant, and the shape of the bass bridge is simplified as a simple plate. The formulation of the discretized eigenvalue problem is:

\[
Kq = \omega^2 Mq
\]

where \(K\) is the elastic rigidity, \(q\) a eigenvector of a mode, \(\omega\) the associated modal pulsation and \(M\) the mass matrix of the actual system. Adding a simply supported boundary condition in equation 1, we find the modal shapes presented in figure 2. The effects of simple boundary conditions in the first eigenfrequencies of the soundboard are presented in table II.

FIG. 2. Shapes of the first seven modes of the piano soundboard with fixed boundaries

These results are the classical flexuring modes, close to the literature ones. However, H. Suzuki\(^7\) found an extra mode with a geometry close to the second mode but a little larger. He explains this mode by the presence of the rim in the experiment, and calls it the "rim mode". But the model of the present paper does not consider the rim and this mode is not observed. The other modes are close to his results. Table II gives the influence
of the boundary conditions on the modal frequencies. Further in this paper, we use a simply supported boundary condition, to simply model the rim possible displacement and the diminution of thickness of the spruce plate close to the rim.

Crown modeling

The initial crown of the soundboard is also implemented into the finite element model. The making process to obtain crown depends on the maker and consists in gluing the ribs and using the humidity. Because of the too large complexity of these processes, the crown is only modeled as a geometric change, excluding the induced prestress field resulting from the process.

The crown is incurred by solving a harmonic scalar problem of the soundboard on the mesh with a constant source in the whole domain and Dirichlet boundary conditions. We then get the altitude at each point of the plate through a smooth function of the coordinates with a zero value on the boundary. The crown level is characterized by the altitude of the top of the crown, named \( b \) and expressed as a multiple of the thickness. A modal calculus gives the evolution of the frequencies and the modal shapes with \( b \). The figure 3 shows an increase of all frequencies for values above about \( b > 1 \). A cross between the first two modes is observed for \( b = 5 \) due to the geometric changes.

Downbearing modeling

The downbearing is modeled as a vertical force per unit length between 0 and 5000 N/m applied on the two bridges, perpendicular to the soundboard. The value of 5000 N/m is approximately the double of the maximum force used in piano making. The loading parameter is finally the maximum displacement of the soundboard induced by the load, denoted \( \lambda \).
Linearized or geometrically nonlinear model?

The downbearing is modeled using the prestress theory. The Hamilton principle for a soundboard subjected to an initial stress $\sigma_0$ (due to the downbearing on the two bridges here) is (see$^{10}$):

$$\delta \int_{t_1}^{t_2} (T^* - V_{\text{int}}^* - V_g^* - V_{\text{ext}}^*) dt = 0$$  \hspace{1cm} (2)

$$\delta u_i^*(t_1) = \delta u_i^*(t_2) = 0$$  \hspace{1cm} (3)

where

$$T^* = \int_{\Gamma^*} \rho^* \dddot{u}_i^* \dddot{u}_i^* dV \text{ the kinetic energy}$$  \hspace{1cm} (4)

$$V_{\text{int}}^* = \int_{\Gamma^*} C_{ijkl}(\varepsilon_{ij}^{(1)} \varepsilon_{kl}^{(1)}) dV \text{ the linear strain energy}$$  \hspace{1cm} (5)

$$V_g^* = \int_{\Gamma^*} \sigma_{ij}^0 \dddot{e}_{ij}^{(2)} dV \text{ the geometric strain energy due to the downbearing}$$  \hspace{1cm} (6)

$$V_{\text{ext}}^* \text{ the potential of the external dynamical loading}$$  \hspace{1cm} (7)

$$u_i^* \text{ the displacement during the interval of time } [t_1, t_2]$$  \hspace{1cm} (8)
where $\Gamma^*$ is the prestressed soundboard, $\sigma_{ij}^0$ is the initial stress field due to downbearing, $\rho^*$ is the density $\varepsilon^{(1)}_{ij} = \frac{1}{2}(\frac{\partial u^*_i}{\partial x_j} + \frac{\partial u^*_j}{\partial x_i})$ and $\varepsilon^{(2)}_{ij} = \frac{1}{2}(\frac{\partial u^*_m}{\partial x_i} \frac{\partial u^*_m}{\partial x_j})$ are respectively the linear part and the quadratic part of the strain field, $C_{ijkl}$ is the elasticity tensor.

Using a finite element method, the discretized Hamilton principle is

$$\delta \int_{t_1}^{t_2} \left( \frac{1}{2} \dot{q}^T M \dot{q} - \frac{1}{2} q^T K q - \frac{1}{2} q^T K_g q + q^T g \right) dt = 0$$

(9)

where $M$ is the mass matrix, $K$ is the stiffness matrix $K_g$ is the geometric stiffness matrix, $g$ is the dynamical load vector and $q$ are the generalized coordinates of the problem.

The discretized equations of the motion are:

$$(K + K_g)q + M \ddot{q} = g(t)$$

(10)

and the discretized eigenvalue problem:

$$(K + K_g)q = \omega^2 M q$$

(11)

where $\omega$ are the eigenpulsations of the structure.

The shape of the soundboard $\Gamma^*$, the natural rigidity $K$, the mass $M$ and the geometric rigidity due to downbearing $K_g$ may be calculated using two approaches: a linearized one, usually used in engineering, and a nonlinear one which considers static large displacements. The first one consists to suppose that the static displacements are small and the geometric rigidity is then proportional to the loading parameter $\lambda$:

$$K_g = \lambda K_g^*$$

(12)

where $K_g^*$ is the geometric rigidity obtained under a unitary load. Equation 11 becomes:

$$(K + \lambda K_g^*)q = \omega^2 M q$$

(13)

This approximation drives to fast calculus, because only one calculus of $K_g^*$ is needed. But it gives erroneous results when the displacements are large enough.

In the case of large displacements, an iterative method with an updated geometric rigidity $K_g$ and shape of the soundboard $\Gamma^*$ at each step of calculus are necessary. The Newton-Raphson method has to be used.
FIG. 4. Evolution of eigenfrequencies with the transverse loading for an initial crown $b = 1$ with linearized (dashed) and nonlinear prestresses approaches (non-dashed) (left) and a zoom on the first frequency (right)

Figure 4 shows the evolution of the first seven eigenfrequencies with these two methods, for the example where $b = 1$ (top of the crown = 8mm). Significant differences occur, particularly when the displacements become about the thickness of the plate (8mm), that is large displacements. The initial crown has also effects on these differences in frequency (see figure 5). When $b = 2$ (top of the crown = 16mm), 25% of differences occurs for $\lambda$=10mm (1.125 times the thickness). The linearized prestress model seems to be unacceptable in this case and a complete nonlinear one is then chosen.

**Effect of downbearing on modal frequencies**

As known experimentally, the modal frequencies are noticeably modified by the downbearing. In the case of an initially flat soundboard, the eigenfrequencies are only increasing with downbearing. These results are presented in figure 6 and are in a good qualitative concordance with the experimental measurements. But when $b = 1$, a new situation is observed numerically (see figure 4). The first frequency primary decreases then increases for the transverse displacement $\lambda$ about 3.5mm (about half of the thickness). The first de-
FIG. 5. Effect of the initial crown (value of $b = 0, 0.5, 1, 1.5, 2$) on the relative differences between linearized and nonlinear prestresses approaches for the first eigenfrequency $((f_{\text{nonlinear}} - f_{\text{linear}})/f_{\text{nonlinear}})$

creasing of frequency comes from the global compression introduced by the downbearing. In this case, the effects of downbearing are in opposition with the initial crown. When $b > 1$ (figure 6), the first frequency only decreases, the upper range of downbearing 5000N/m is not sufficient to induce a global traction and then an increase of the eigenfrequencies. These results are more or less identical for all modal frequencies, but with different variations.

We tried to fit the evolution of the frequency as a polynomial function of the loading parameter. Calculations have shown the second degree function of the equation 14 is sufficient to obtain an excellent fit as one can see table III.

$$
\left(\frac{\omega}{\omega_0}\right)^2 = k_0 + k_1 \frac{\lambda}{e} + k_2 \left(\frac{\lambda}{e}\right)^2
$$

(14)

where $\omega$ is the eigenpulsation, $\omega_0$ the eigenpulsation without downbearing $\lambda$, and $e$ the thickness of the plate.

The values of the interpolating coefficients for different crowns are presented table III. This function models very well the relation between the square of the pulsation $\omega^2$ and $\lambda$. 

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FIG. 6. Effects of downbearing on a flat soundboard (left) and the case $b = 2$ (right), using a geometric nonlinearity calculus

for all values of $b$. The value of $k_0$ has to be equal to 1 because it gives the relation $\omega = \omega_0$ without downbearing ($\lambda = 0$). The values of $k_1$ gives the initial slope. The sign of $k_1$ seems to be independent of the eigenmode and is negative for $b \neq 0$ (due to the opposition between downbearing and crown). The quadratic term $k_2$ is never negligible relatively to $k_1$, which characterizes the importance of taking into account the large displacements in the model.

It is important to note that the crown $b$ effects these values. That means the downbearing effects are very dependent of the initial crown.

**Effects on mobility**

*The question of dissipation*

The question of losses has not been considered in the previous calculus. But the losses largely characterize the soundboard, which acts as a strings vibration filter. There are many sources of losses in the soundboard because of the material, the radiation and the assembling with rim, ribs, bridges and finally strings. H. Suzuki\(^7\) expressed these losses as modal damping factor and found a value of 0.064 for the first mode and about 0.02 for the others.
**Examples of mobility**

The experimental values of the damping (see H. Suzuki\(^7\)) are introduced in complement of the finite element results. The transverse mobility of the bridge \(Y\) is easy to express as \(Y = v/F\) where \(v\) is the vertical velocity and \(F\) is the vertical applied force. The mobility for the middle of the treble bridge is presented figure 7.

The general shape of the mobility is similar to the results of the experimental literature in the range of \([0, 450\text{Hz}]\) (see for example H. Suzuki\(^7\)). One can see the noticeable effects of the downbearing when \(b = 0\), as it has been shown experimentally by K. Wogram\(^4\). The modifications of frequencies have been explained in a previous section.

![Graphs showing mobility](image)

**FIG. 7.** Mobility on the middle of the treble bridge for \(b = 0\) (left) and \(b = 2\) (right)

The amplitude of the pics of mobility are also modified by downbearing, due to the slight changes in the modal shapes. When \(b = 2\), the mobility differs from the previous case, and the tendencies are inverted.

**Conclusion**

This numerical work describes the changes in the soundboard vibrations due to the downbearing. To be correctly modeled, the downbearing must be calculated with a geomet-
ric nonlinear approach with prestresses (including the hypothesis of large displacements), because a linear model gives erroneous modal behaviour in the range of the making process. Frequencies and modal shapes are affected by this part of piano makers know-how.

The experimental increase of eigenfrequencies is numerically obtained for an initially flat soundboard. However, the initial crown modifies largely these effects, introducing an initial decrease of eigenfrequencies, and has to be considered for future experiments.

This work is part of a research about consequences of making processes on the quality of musical instruments. Even if this paper is focused on the mechanical effects of the downbearing, the objective is to link these effects to changes in perceivable timbre. An interesting way to create this link is to include the results of this paper into some synthesis of piano sounds.

Acknowledgments

The authors want to thank R. Bresson and F. Conti, students of the Ecole Nationale des Ponts et Chaussées for their contributions in the finite element mesh.

References


TABLE I. Mechanical properties of a spruce soundboard, see\textsuperscript{6}

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( E_x )</th>
<th>( E_y )</th>
<th>( \nu_{xy} )</th>
<th>( G_{xy} )</th>
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<td>392 kg.m\textsuperscript{-3}</td>
<td>11.5 GPa</td>
<td>0.47 GPa</td>
<td>0.005</td>
<td>0.5 GPa</td>
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TABLE II. Frequencies of the first seven modes of the piano soundboard for different boundary conditions

<table>
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<th>Mode</th>
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<th>fixed (Hz)</th>
<th>free (Hz)</th>
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<tr>
<td>1</td>
<td>93.7</td>
<td>75.9</td>
<td>34.0</td>
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<td>2</td>
<td>169</td>
<td>143</td>
<td>52.4</td>
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<td>3</td>
<td>216</td>
<td>186</td>
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<td>4</td>
<td>273</td>
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<td>5</td>
<td>299</td>
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<td>356</td>
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<td>7</td>
<td>383</td>
<td>339</td>
<td>147</td>
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TABLE III. Coefficients of the interpolating quadratic functions for $b = 0$, $b = 1$ and $b = 2$

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<th>$k_1$</th>
<th>$k_0$</th>
<th>error max</th>
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<td>0.992</td>
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<td>0.999</td>
<td>2.10.10^{-3}</td>
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<tr>
<td>3</td>
<td>0.0497</td>
<td>0.0796</td>
<td>1.00</td>
<td>4.43.10^{-4}</td>
</tr>
<tr>
<td>4</td>
<td>0.0458</td>
<td>0.102</td>
<td>1.00</td>
<td>9.04.10^{-4}</td>
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<tr>
<td>5</td>
<td>0.0425</td>
<td>0.0625</td>
<td>1.00</td>
<td>1.10.10^{-3}</td>
</tr>
<tr>
<td>6</td>
<td>0.0435</td>
<td>0.0388</td>
<td>1.00</td>
<td>1.00.10^{-3}</td>
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<tr>
<td>7</td>
<td>0.0156</td>
<td>0.0675</td>
<td>0.998</td>
<td>2.00.10^{-3}</td>
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<table>
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<tr>
<th>Mode</th>
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<th>$k_1$</th>
<th>$k_0$</th>
<th>error max</th>
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<table>
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<th>$k_1$</th>
<th>$k_0$</th>
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<td>1.07.10^{-4}</td>
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<tr>
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<td>1.00</td>
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<td>4.80.10^{-5}</td>
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</tbody>
</table>
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